EXTENDING NUMBER SENSE TO FRACTIONS – 3RD-5TH Webinar #2: Adding & Subtracting Fractions

ALASKA MATHEMATICS STANDARDS

Content Standards

Standards of Mathematical Practice (SMPs)

STANDARDS FOR MATHEMATICAL PRACTICE
1. I can make sense of problems and persevere in solving them.
2. I can use numbers, words, and reasoning habits to help me make sense of problems.
3. I can construct viable arguments and critique the reasoning of others.
4. I can recognize math in everyday life and use math I know to solve problems.
5. I can use appropriate tools strategically.
6. I can be precise when solving problems and clear when communicating my ideas.
7. I can look for and make use of structure. I can see one way of writing a number and another way of writing the same number.
8. I can notice patterns and find more general methods and “short cuts.”
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6. I can be precise when solving problems and clear when communicating my ideas.
7. I can look for and make use of structure: I can see and understand how numbers are organized and composed as parts and wholes.
8. I can notice patterns and find more general methods and "short cuts".

REPRESENTATION IN MATH

Concrete  Pictorial  Abstract

FRACTIONS IN THE NEW STANDARDS

3rd Grade
- 3.NF: Develop understanding of fractions as numbers.
  (limited to fractions with denominators 2, 3, 4, 6, and 8)

4th Grade
- 4.NF: Extend understanding of fraction equivalence and ordering.
- 4.NF: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- 4th Grade: Limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, & 100.

5th Grade
- 5.NF: Use equivalent fractions as a strategy to add and subtract fractions.
- 5.NF: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
- 5th Grade: Limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, & 100.
3 WAYS TO REPRESENT FRACTIONS

- Area Model
- Set Model
- Linear Model

FRACTIONS AS NUMBERS

What size piece are we working with?
How many equal pieces is the whole split into?

The whole is 1 candy bar.

FRACTIONS AS NUMBERS

1 whole

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 / 5</td>
<td>2 / 5</td>
<td>3 / 5</td>
<td>4 / 5</td>
<td>1</td>
</tr>
</tbody>
</table>

Number of those pieces:

2 / 3
SENSE-MAKING VS. ANSWER-GETTING

- What is the whole?
- What size pieces do we have?
- How many of those pieces?
- Estimate (is the answer reasonable?)
- How can I best represent this problem?

FRACTIONS IN THE NEW STANDARDS:

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• Add/Subtract
• Like Denominators
• Fractions w/denominators: 2, 3, 4, 5, 6, 8, 10, 12, 100
. 4.NF.3. Understand a fraction $a/b$ with $a > 1$ as a sum of fractions $1/b$. 

$$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

5 copies of $1/6$. 

UNIT FRACTIONS AS BUILDING BLOCKS
What are some other ways to represent 5/6 as 5 copies of 1/6?

4.NF.3.a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

\[
\frac{5}{6} = \frac{1}{6} + \frac{4}{6}
\]

5 sixths = 1 sixth + 4 sixths

\[
\frac{3}{8} + \frac{2}{8} = \frac{5}{8}
\]

3 eighths + 2 eighths = 5 eighths
**POTENTIAL ROADBLOCK!**

Visual Model

\[
\frac{2}{3} + \frac{1}{3} = \frac{3}{6}.
\]

**Be mindful of the whole.**

\[
2 \text{ thirds} + 1 \text{ third}.
\]

Estimate

\[
> \frac{1}{2}
\]

\[
\frac{1}{2}
\]

It is impossible to start with more than \(\frac{1}{2}\), add a positive amount, and end up with \(\frac{1}{2}\).

---

**Working Towards the Standard Algorithm**

\[
\frac{7}{5} + \frac{4}{5} = \frac{11}{5}
\]

7 fifths + 4 fifths = 11 fifths

---

**Subtraction - Take Away**

\[
\frac{5}{6} - \frac{4}{6} = \frac{1}{6}
\]

\[
\frac{5}{6} - \frac{4}{6} = \frac{1}{6}
\]
Subtraction – Missing Addend

\[
\frac{5}{6} - \frac{4}{6} = \frac{1}{6},
\]

\[
\frac{4}{6} + ? = \frac{5}{6},
\]

\[
\frac{4}{6} + \frac{1}{6} = \frac{5}{6}.
\]

4.NF.3.b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation.

\[
\frac{5}{6} = \frac{1}{6} + \frac{4}{6},
\]

\[
\frac{5}{6} = \frac{2}{6} + \frac{3}{6}.
\]

Decomposing & Composing Fractions

\[
\frac{5}{6} = \frac{3}{6} + \frac{2}{6}.
\]
4.NF.3.c Add and subtract **mixed numbers** with like denominators (e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition & subtraction.)

**Mixed Numbers & Improper Fractions**

\[3 \frac{1}{4} = 3 + \frac{1}{4}\]
MIXED NUMBERS & IMPROPER FRACTIONS

When is having a mixed number more helpful than having improper fractions?

When is having an improper fraction more helpful than having mixed numbers?
4.NF.3.c Add and subtract mixed numbers with like denominators (e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition & subtraction.

**Replacing with Equivalent Fractions**

\[
3 \frac{1}{4} + 2 \frac{2}{4}
\]

\[
\frac{3}{4} + \frac{2}{4} + \frac{2}{4} = \frac{7}{4}
\]

**Using Properties of Operations**

\[
3 \frac{1}{4} + 2 \frac{2}{4}
\]

\[
3 + \frac{1}{4} + 2 + \frac{2}{4}
\]

\[
3 + 2 + \frac{1}{4} + \frac{2}{4}
\]

\[
5 + \frac{3}{4}
\]

\[
5 \frac{3}{4}
\]
REGROUPING?

\[ \begin{align*}
2 \frac{1}{6} - 1 \frac{5}{6} &= \frac{2}{6} \\
&= \frac{2}{6} - \frac{15}{6} \\
&= \frac{2-15}{6} \\
&= \frac{-13}{6}
\end{align*} \]

REGROUPING - START WITH WHOLE #: FRACTION

\[ \begin{align*}
3 \frac{2}{5} &= \frac{17}{5}
\end{align*} \]

DECOMPOSING

\[ \begin{align*}
6 \frac{2}{8} - 4 \frac{5}{8} &= \frac{46}{8} - \frac{37}{8} \\
&= \frac{46-37}{8} \\
&= \frac{9}{8}
\end{align*} \]
4.NF.3.D Solve word problems involving addition & subtraction of fractions referring to the same whole and having like denominators (e.g., by using visual fraction models and equations to represent the problem).

During recess, Marisa threw me \( \frac{7}{12} \) of the way across the football field. Marcus threw me \( \frac{6}{12} \) of the way. How much further did Marisa throw than Marcus?
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### Fractions in the New Standards

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#### 5.NF.5: Use equivalent fractions as a strategy to add and subtract fractions.

- Unlike Denominators
- Fractions with any denominator

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Answer Getting vs. Sense-Making

\[
\frac{1}{2} + \frac{1}{3} = \frac{2}{5} ?
\]
5.NF.1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$.

(In general, $\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}$.)

**CONNECT: CONCRETE -- SYMBOLIC**

\[
\begin{align*}
\frac{3}{4} \times 3 & \quad + \quad \frac{2}{3} \times 4 \\
\frac{9}{12} & \quad + \quad \frac{8}{12} \\
\frac{17}{12} & \quad + \quad 1 + \frac{5}{12}
\end{align*}
\]
THINK, (PAIR), SHARE

Write down 3 different pairs of fractions that will add to a result a little more than 5. DO NOT actually do the addition!

Explain your reasoning:
THINK; (PAIR), SHARE:
• Write down 3 different pairs of fractions that will add to a result a little less than 5. DO NOT actually do the addition!
• Explain your reasoning.

5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and check the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.

WORD PROBLEMS WITH FRACTIONS:
Ludmilla and Lazarus each have a lemon. They need a cup of lemon juice to make hummus for a party. Ludmilla squeezes 1/2 cup from hers and Lazarus squeezes 2/5 of a cup from his. How much lemon juice do they have? Is it enough?

\[ \frac{1}{2} + \frac{2}{5} = \]
\[ \frac{1}{2} + \frac{2}{5} \]

Estimate:
\[ \frac{1}{2} + \approx \frac{1}{2} \]
\[ \approx 1 \]

Since \( \frac{3}{5} < \frac{1}{2} \), then I know it’s just less than 1.

Calculate:
\[ \frac{1}{2} + \frac{2}{5} = \frac{9}{10} \]
\[ \frac{1}{2} + \frac{2}{5} = \frac{3}{7} \]

Check Reasonableness:
\[ \frac{9}{10} \text{ is just less than } \frac{10}{10}, \text{ so it is just less than } 1. \]
\[ \frac{3}{7} \text{ is way less than } \frac{7}{7}, \text{ so it is way less than } 1. \]
In fact, it is less than \( \frac{1}{2} \).

5. NF Do These Add Up?
For each of the following word problems, determine whether or not \( \left( \frac{1}{2} + \frac{1}{3} \right) \) represents the problem. Explain your decision.

a. A farmer planted \( \frac{1}{2} \) of his forty acres in corn and another \( \frac{1}{3} \) of his land in wheat. Taken together, what fraction of the 40 acres had been planted in corn or wheat?

b. Jim drank \( \frac{1}{2} \) of his water bottle and John drank \( \frac{1}{3} \) of his water bottle. How much water did both boys drink?

c. Allison has a batch of eggs in the incubator. On Monday \( \frac{1}{2} \) of the eggs hatched. By Wednesday, \( \frac{2}{3} \) more of the original batch hatched. How many eggs hatched in all?

d. Two fifths of the cross-country team arrived at the weight room at 7 a.m. Ten minutes later, \( \frac{3}{10} \) of the team showed up. The rest of the team stayed home. What fraction of the team made it to the weight room that day?
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What are some key take-aways?

Go to: www.teachingchannel.org

“Sign Up” with your school email address. There is no cost. Great resource!

If you take the credit class we will use the Teaching Channel Teams platform.

Resource for this webinar: Fractions with Borrowing

https://www.teachingchannel.org/videos/fractions-with-borrowing

Agree?, Argue?, Aspire?
Thank you!